M.Sc. - Mathematics
I Semester End Examination - May 2022
Discrete Mathematics

Course Code: MM105T **QP Code: 11005**
Time: 3 hours **Total Marks: 70**

Instructions: 1) **All** questions carry **equal** marks.

2) Answer **any five** full questions.

1. a) Using the rules of inferences, test the validity of the argument:

   If I get distinction in MSc, then I will join for a PhD.
   If I don't get distinction in MSc, then I will work in a company.
   If I work in a company, then I will get a salary.
   Therefore, If I don't join for PhD, then I will get a salary.

   b) Define the conjunctive normal form and obtain the same for the compound proposition $(p \wedge q) \vee (\sim p \wedge r)$.

   c) Explain the direct proof method and use it to prove that product of two even integers is an even integer. (5+5+4)

2. a) 35 children of a class draw a map. 26 children use red colour and some children use yellow colour. If 19 use both the colours, find the number of children who used the yellow colour.

   b) Prove that if 30 dictionaries in a library contain a total of 61,327 pages, then atleast one of the dictionaries must have at least 2045 pages.

   c) In how many different ways can eight identical cookies be distributed among three distinct children if each child receives at least two cookies and not more than four cookies?

   d) How many ways are there to pick two different cards from a standard 52-card deck such that (i). The first card is an Ace and the second card is not a Queen?
   (ii.) The first card is a spade and the second card is not a Queen? (3+3+4+4)

3. a) Model the rabbit population as recurrence relation and solve it explicitly.

   b) Solve the recurrence relation $a_{n+2} + 3a_{n+1} + 2a_n = 3^n$ for $n \geq 0$, with initial conditions $a_0 = 0,\ a_1 = 1$.

   c) Using the generating functions, solve the recurrence relation $a_n - 3a_{n-1} = 2$ for $n \geq 1$ with the initial condition $a_0 = 1$. (5+5+4)

4. a) Given a set A with $|A| = n$, and a relation R on A, let M be the relation matrix of R, then prove the following:
   i. R is reflexive iff $I_n \leq M$.
   ii. R is antisymmetric iff $M \cap M^T \leq I_n$.

   b) Let $A = \{1,2,3,4,6,12\}$. On A, define the relation R by aRb iff a divides b. Prove that R is a partial order on A. Draw the Hasee diagram for this relation.

   c) Define reflexive and symmetric closure. Find the transitive closure of the relation

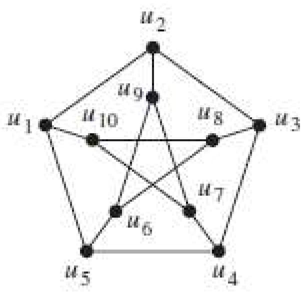
   $R = \{(1,2),(2,3),(3,1)\}$ defined on set $A = \{1,2,3\}$ using Warshall's algorithm.

   (5+5+4)

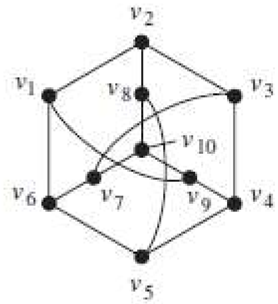
5 a) State and prove the first theorem in graph theory, further show that in any graph G, the number of vertices in odd degree is even.

b) Define a component of a graph. Prove that a simple graph with p vertices and k components have size atmost $\frac{(p-k)(p-k+1)}{2}$ edges.

c) Define isomorphism of graphs. Verify whether the following graphs are isomorphic or not



G_1



G_2

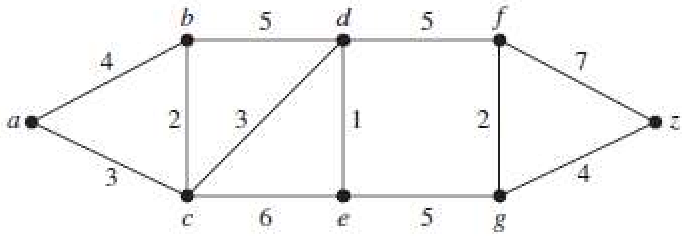
(4+5+ 5)

6 a) Define a walk and a path. Prove that every closed walk contains an odd cycle.

b) Define radius, diameter, center, periphery of a graph G. Prove that

$rad(G) \leq diam(G) \leq 2rad(G)$

c) Applying Dijkstra's algorithm. Find the shortest-path between 'a' and 'z' for the weighted graph (4+5+5)



7 a) A connected graph G contains Eulerian trail if and only if it has exactly two vertices of odd degree.

b) If G is a simple graph with $p \geq 3$, show that $deg(u) + deg(v) \geq p$ for every pair of non adjacency vertices then G is Hamiltonian.

c) Let e be an edge in a connected graph G. Prove that e is a bridge if and only if e doesnot lie on a cycle. (5+5+4)

8 a) Define a tree. Prove that a graph G is a tree T if and only if every pair of vertices are connected by a unique path.

b) Prove that every tree has T has atleast two pendent vertices.

c) Define minimal spanning tree. Explain Kruskal's algorithm with an example. (5+4+5)

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